

Section 2.3

An argument is a sequence of statements.

The last statement is called the conclusion.

All the preceding statements are called the assumptions or hypotheses or premises.

(The conclusion usually starts with “Therefore” or “So”.)

An argument is valid if and only if

the conclusion is true in all cases where the assumptions are true.

So, an argument is invalid if and only if

there is a situation in which the assumptions are true, but the conclusion is false.

**The validity of an argument is NOT simply determined by
whether the conclusion is true or false by itself.**

To determine if an argument is valid or invalid,

construct a truth table with columns for each assumption and the conclusion.

Highlight all rows in which all assumptions are true. (These are called the critical rows.)

If the conclusion is true in all those rows, the argument is valid.

A valid argument is a logical process by which we reach a conclusion.

The process itself assumes correct inputs (true statements),

and gives correct outputs (true conclusions).

If the process is given incorrect inputs (false statements),

there is no guarantee whether the outputs are correct or not.

A computer science parallel for the difference between

a valid argument versus a true conclusion:

Suppose a class is asked to write a program to read an arithmetic expression

of the form “ $a \otimes b$ ” (where a and b are numbers and \otimes is either $+$, $-$, \times or \div)

from a user’s input, and evaluate the expression.

John writes the program so that it simply says “1” no matter what the user inputs.

He runs a test with the inputs “ $6 - 5$ ”, “ $9 \div 9$ ”, “ 2×0.5 ” and “ $-7 + 8$ ”.

The answers are all correct.

This is parallel to an invalid argument (the program)

with a true conclusion (the answers).

Jane writes the program to work exactly as required if the user inputs a correct arithmetic expression.

She runs the test with the input “ $\text{Dog} \wedge \text{Cat}$ ”.

Her program crashes.

This is parallel to a valid argument (the program)

with a false conclusion (the crash) due to a false assumption (the input).

ARGUMENT #0

If today is Wednesday, then Math 22 meets today.

Math 22 meets today.

Therefore, today is Wednesday.

ARGUMENT #1

If today is Wednesday, then this is Math 43.

Today is Wednesday.

Therefore, _____.

ARGUMENT #2

If you study hard, then you will do well.

You didn't do well.

Therefore, _____.

ARGUMENT #3

Today is Wednesday.

Therefore, _____ or this is Math 43.

ARGUMENT #4

Today is Wednesday and this is Math 22.

Therefore, _____.

ARGUMENT #5

Today is Wednesday.

This is Math 22.

Therefore, _____.

ARGUMENT #6

Today is Wednesday or this is Math 43.

This is not Math 43.

Therefore, _____.

ARGUMENT #7

If today is Wednesday, then Math 22 meets today.

If Math 22 meets today, then I cannot sleep in until noon.

Therefore, _____.

ARGUMENT #8

Today is Wednesday or today is Thursday.

If today is Wednesday, then Math 22 meets today.

If today is Thursday, then Math 22 meets today.

Therefore, _____.

ARGUMENT #9

If today is Tuesday, then Math 22 meets today.

Today is not Tuesday.

Therefore, Math 22 does not meet today.

“ARGUMENT” # 10

Someone killed Mr Body using a weapon. Assuming all the following statements are true, can you determine who the killer was and what weapon they used ?

If Miss Plum did not use the gun, then Mrs Red did not use the knife and Ms Scarlet did not use the wrench.

Either Mr Violet did not use the hammer or Mr Yellow used the poison.

If Mr White did not use the candlestick, then Ms Scarlet used the wrench.

If Mr Violet did not use the hammer, then Miss Plum did not use the gun.

Mr Yellow did not use the poison.

NOTE:

The argument

Statement 1

Statement 2

Statement 3 ...

Statement n

Conclusion

is valid if and only if the compound statement

$(\text{Statement 1} \wedge \text{Statement 2} \wedge \text{Statement 3} \wedge \dots \wedge \text{Statement n}) \rightarrow \text{Conclusion}$

is a _____.